

SNR of a ideal ADC

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From the first presentation (1982) by Philips and Sony for the first HI-FI digital CD, many amateurs have learned (made know also ARRL) that:

$$\text{ADC's SNR (dB) = 1.76 dB + (N * 6.02)dB} \quad (1)$$

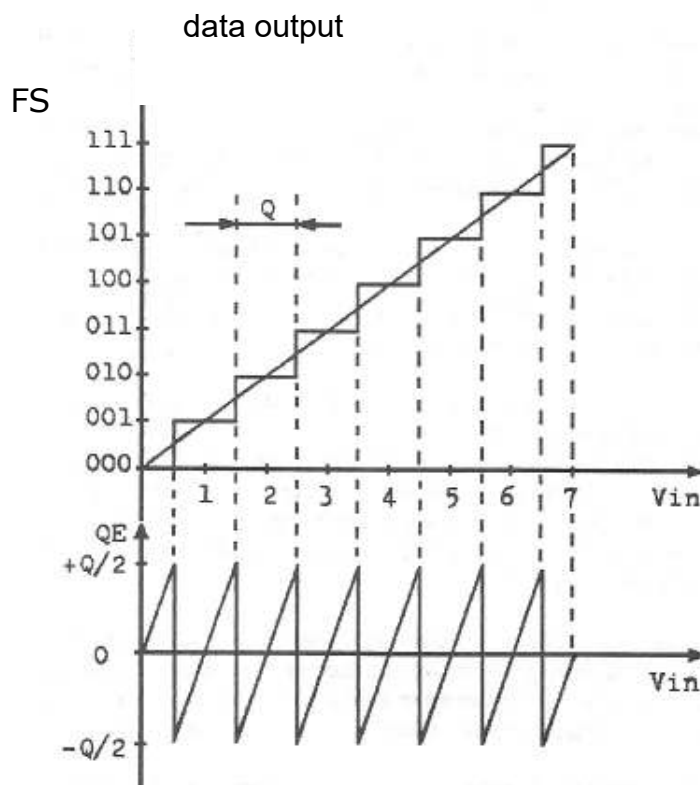
Where N is the bit's number.

But the our Radioamateur Handbook has never said the why of numbers 6.02 and 1,76. As are they obtained?

Let's start by saying that: The SNR of any system and thus also for the ADC is equal :

$$SNR = \frac{rmsV_{input}}{rmsV_{noise}} = \quad (2)$$

In the figure for simplicity a 3 bits ADC with FS $V_{in} = 7 V_{pp}$



You can see that an input voltage of 2 volts pp corresponds has a binary output 010. The 2 volt analog value is halfway the two voltage level 1.5 and 2.5 Vpp.

Therefore any input value between 1.5 and 2.5 Vpp, will be converted, at the same output value: 010.

The magnitude of the error of quantization, Q , is equal to the difference between the value assumed by the input sample and the corresponding output value.

If the input signal is made continuously vary (a sinusoidal signal) through to full scale, FS, of values and subtracted from time to time to the corresponding output value we get a error voltage of the form called: saw tooth.

The error, Q , will be smaller with higher will be the number of bits.
(see Tab.1)

The peak of maximum quantization error is equal to $Q/2$. The maximum peak to peak value is equal to $Q = (Q/2 - (-Q/2))$

The rms voltage of saw tooth signal is:

$$rmsQ = \frac{Q_{pp}}{2 \cdot \sqrt{3}} \quad (3)$$

This error voltage is uncorrelated from the input signal ADC and can be seen as a noise. It is called "quantization's noise".

The voltage rms of sinusoidal signal input ADC is :

$$rmsV_{input} = \frac{V_{pp}}{2 \cdot \sqrt{2}} = \frac{2^N \cdot Q}{2 \cdot \sqrt{2}} \quad (4)$$

With a un ideal ADC (DNL=0, ADC input thermal noise = 0 and jitter OL = 0

From eq. (2) the SNR (dB) will be:

$$idealSNR(dB) = 20 \cdot \text{Log} \frac{2^N \cdot Q}{2 \cdot \sqrt{3}} = (6.02 \cdot N) + 1.76. \quad (5) = (1)$$

Where N is number bits. Applying eq. (5) we obtain:

Tab. 1

N BITS	RESOLUTION	VALUE OF ERROR Q	SNR
	$1/2^N$	For a FS= 2.5 Vpp	dB
10	1/ 1024	2.44 mV	61.96
12	1/ 4096	0.61 mV	74
14	1/ 16384	0.15 mV	86.04
16	1/ 65536	38.1 uV	98.08
18	1/262144	9.5 uV	110.16

Acronyms used.

ADC -Analog to Digital Converter

DNL- Differential non linearity.

FS - Full Scale ADC

rms - root mean square.

SNR- Signal to Noise Ratio

Vpp- Volt peak to peak

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